Flat Springs

SPRINGS & THINGS
Manufactured To Last

General Data

The term "flat springs" covers a wide range of springs or stampings fabricated from flat strip material which, on being deflected by an external load, will store and then release energy. Only a small portion of a complex shaped stamping may actually be functioning as a spring. For the purposes of design, that portion which acts as a spring may often be considered as an independent simple spring form, while the rest of the part is temporarily ignored.

Cantilever and Beam Springs

In comparing the load and stress equations for both types of springs, it can be seen that an equal active length (L) of identical cross-section (b and t) produces 16 times the load in a simple beam spring as in a cantilever spring. However, the stress in the simple beam spring is four times that of the cantilever spring for a given deflection.

The differences between the two types of springs cancel out when they are compared in terms of volume of active spring material. A simple beam spring can be designed with greater length and decreased thickness so it will have the same load, same deflection, same stress and same volume of material as a given cantilever spring.

Because load (P) varies as the third power of the thickness (t), the flat spring material should have minimum variation in thickness. Load also varies as the third power of length. (See Table 1, below.)

Figure 1.
CANTILEVER SPRING

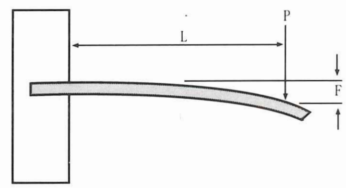
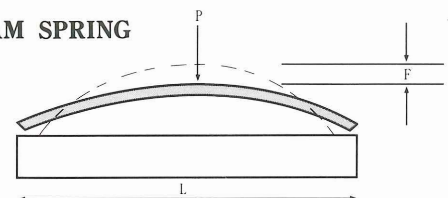


Figure 2.
SIMPLE BEAM SPRING



Design Formulas

The formulas for load (P) and bending stress (S) used in cantilever and simple beam springs are as follows:

Cantilever Spring

(1) $P = \frac{EFbt^3}{4L^3}$ lb. (N)

(2) $S = \frac{3EFt}{2L^2} = \frac{6PL}{bt^2}$ psi (MPa)

Simple Beam Spring

(3) $P = \frac{4EFbt^3}{L^3}$ lb. (N)

(4) $S = \frac{6EFt}{L^2} = \frac{3PL}{2bt^2}$ psi (MPa)

where

P = Load, lb. (N)

E = Modulus of elasticity, psi (MPa)

F = Deflection, in. (mm)

t = Thickness of material, in. (mm)

L = Active spring length, in. (mm)

b = Width of material, in. (mm)

S = Design bending stress, psi (MPa)

*In some manuals h instead of t is used for thickness.

Design Method

The design of the many special types of flat springs is covered thoroughly in many standard texts and technical articles and will not be covered in this handbook. Use of the basic flat spring formulas for the cantilever and simple beam types of springs is demonstrated in the examples given here.

The design method for the two basic types of flat springs—cantilever and simple beam springs—is quite easy in comparison to designing helical compression, extension, and torsion springs. While flat springs may appear to be extremely complex in shape, the engineer need only consider those parts which are active in operation. It is likely, then, that the active section is no more than an ordinary cantilever or simple beam spring.

Table 1.
FLAT SPRINGS
Strip Thickness Tolerances, Tempered and Untempered

Thickness Range In. (mm)	Tolerance, In. (mm) ±	Tolerance, In. (mm) Total
0.125 - 0.063 (3.18) - (1.60)	0.00200 (0.051)	0.0040 (0.102)
0.062 - 0.040 (1.59) - (1.02)	0.00150 (0.038)	0.0030 (0.076)
0.039 - 0.029 (1.01) - (0.74)	0.00100 (0.025)	0.0020 (0.051)
0.028 - 0.020 (0.73) - (0.51)	0.00075 (0.019)	0.0015 (0.038)
0.019 - 0.007 (0.50) - (0.18)	0.00050 (0.013)	0.0010 (0.025)
Under 0.007 (0.18)	0.00030 (0.008)	0.0006 (0.015)

Design Examples

EXAMPLE 1

Design a cantilever spring for a magazine pawl in a textile machine. The material is AISI 1074 steel. The active length is L = 1.25 in. (31.75 mm) and the width is b = 0.250 in. (6.35 mm). It must exert a load of P = 5.5 oz. (1.53 N) on the pawl and must deflect F = 0.250 in. (6.35 mm) as the ratchet teeth pass by.

When a first load is given and a further deflection is required, it is a good rule to assume that deflection to the first load will be the same as the deflection between the first and the second loads. Therefore, the load in the second deflected position is twice the load in the first deflected position. In this example, then, the maximum load will be P = 0.687 lb. (3.06 N) and the maximum deflection will be F = 0.500 in. (12.7 mm).

Thickness

Transpose formula 1 for cantilever spring to solve for thickness.

$$t = \sqrt[3]{\frac{4PL^3}{EFb}} = \sqrt[3]{\frac{4(0.687)(1.25)^3}{30 \times 10^6(0.5)(0.25)}} = 0.0113 \text{ in. } (0.29 \text{ mm})$$

Use t = 0.012 in. (0.30 mm)

Stress

Solve for stress using formula 2 for cantilever springs.

$$S = \frac{6PL}{bt^2} = \frac{6(0.687)(1.25)}{(0.25)(0.012)^2} = 143{,}000 \text{ psi } (986 \text{ MPa})$$

The minimum tensile strength of this material will be approximately 220,000 psi (1517 MPa). The elastic limit is about 75 percent of tensile or 165,000 psi (1138 MPa), and the stress for long life should be about 50 percent of tensile or 110,000 psi (758 MPa). The stress of 143,000 psi (986 MPa) for this design would then be suitable only for limited fatigue life, and so the stress must be reduced to 110,000 psi (758 MPa) or lower.

To reduce the stress, either the width or length must be increased or the load or deflection must be decreased. In the above example, assume that the length can be increased from 1.25 in. (31.75 mm) to 1.75 in. (44.45 mm).

Solve for thickness and stress as before.

$$t = \sqrt[3]{\frac{4(0.687)(1.75)^3}{30 \times 10^6(0.5)(0.25)}} = .0158 \text{ in. } (0.40 \text{ mm})$$

Use t = 0.016 in. (0.41 mm)

$$S = \frac{6(0.687)(1.75)}{(0.25)(0.016)^2} = 113{,}000 \text{ psi } (779 \text{ MPa})$$

Since it is close to 110,000 psi (758 MPa) the stress level should give satisfactory fatigue life unless stress concentration or corrosion are factors.

The stress formula 2 should be used with caution because the application requires a maximum deflection of 0.500 in. (12.7 mm) regardless of the load developed by the spring. In the above example, the stress of 143,000 psi (986 MPa) could have instead been reduced by reducing the thickness, but the load also would have been decreased.

EXAMPLE 2

Redesign a simple beam spring which has worked satisfactorily as an anti-rattle device but which must be reduced in cost. The material is Type 302 stainless steel. The dimensions are b = 0.750 in. (19.05 mm.), t = 0.025 in. (0.64 mm), and L = 2.5 in. (63.50 mm.) Free height is 0.250 in. (6.35 mm) and assembled height 0.062 in. (1.57 mm).

Stress

Solve first for stress in the present spring using formula 4.

$$S = \frac{6EFt}{L^2} = \frac{6(28 \times 10^6)(0.187)(0.025)}{(2.5)^2} = 126{,}000 \text{ psi } (869 \text{ MPa})$$

The minimum tensile strength of Type 302 stainless is 185,000 psi (1276 MPa), so that the design stress in bending is 0.75(185,000) = 139,000 psi (958 MPa). However, it is possible to utilize higher stress up to 100 percent of tensile by removing set. In this application the spring can be made to a greater free height, and set can be removed at assembly without adding an operation. Therefore, redesign the spring to an allowable maximum bending stress of 185,000 psi (1276 MPa).

Load

Find the load (P) in the present spring using formula 3.

$$P = \frac{4EFbt^3}{L^3} = \frac{4(28 \times 10^6)(0.187)(0.75)(0.025)^3}{(2.5)^3} = 15.7 \text{ lb. } (69.8 \text{ N})$$

In order to maintain the same load and deflection as the original spring, the ratio of length to thickness must remain the same (t^3/L^3 in formula 3). Substituting L/t = 2.5/0.025 = 100 in formula 4.

$$S = \frac{3P}{2bt}\left(\frac{L}{t}\right) = \frac{150P}{bt} \text{ psi (MPa)}$$

Thickness and Length

Transpose to solve for t using the new value for stress.

$$t = \frac{150P}{bS} = \frac{150(15.7)}{0.75(185{,}000)} = 0.017 \text{ in. } (0.43 \text{ mm})$$

L = 100t = 100(0.017) = 1.7 in. (43.2 mm)

For the calculated dimensions, the weight of the new spring will then be about one-half of the original design.

Tolerances

Loads are influenced by normal variation in strip thickness, particularly in thin highly stressed flat springs. Table 1 gives the tolerances for flat spring materials.